EXAMINATIONS COUNCIL OF ZAMBIA

Examination for General Certificate of Education Ordinary Level

Mathematics

Paper 2

Thursday 1 August 2019

Additional materials:
Answer Booklet
Silent Electronic Calculator (non-programmable)
Geometrical Instruments
Graph paper (3 sheets)
Plain paper (1 sheet)

Time: 2 hours 30 minutes

Instructions to Candidates

Write your name, centre number and candidate number in the spaces provided on the Answer Booklet.
Write your answers and working in the Answer Booklet provided.
If you use more than one Answer Booklet, fasten the Answer Booklets together.
Omission of essential working will result in loss of marks.
There are twelve (12) questions in this paper.

Section A
Answer all questions.

Section B
Answer any four questions.

Silent non-programmable Calculators may be used.
Cell phones are not allowed in the examination room.

Information for Candidates

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 100.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give
the answer to three significant figures. Give answers in degrees to one decimal place.
1 ALGEBRA
Quadratic Equation
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

2 SERIES
Geometric Progression
\[ S_n = \frac{a(1 - r^n)}{1 - r}, (r < 1) \]
\[ S_n = \frac{a(r^n - 1)}{r - 1}, (r > 1) \]
\[ S_\infty = \frac{a}{1 - r} \] for \( |r| < 1 \)

3 TRIGONOMETRY
Formula for \( \Delta \) ABC
\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]
\[ a^2 = b^2 + c^2 - 2bc \cos A \]
\[ \Delta = \frac{1}{2} bc \sin A \]

4 STATISTICS
Mean and standard deviation
Ungrouped data
\[ \text{Mean} (\bar{x}) = \frac{\sum x}{n}, \text{SD} = \sqrt{\left( \frac{\sum (x - \bar{x})^2}{n} \right)} = \sqrt{\left( \frac{\sum x^2}{n} - (\bar{x})^2 \right)} \]
Grouped data
\[ \text{Mean} (\bar{x}) = \frac{\sum fx}{\sum f}, \text{SD} = \sqrt{\left( \frac{\sum f(x - \bar{x})^2}{\sum f} \right)} = \sqrt{\left( \frac{\sum f x^2}{\sum f} - (\bar{x})^2 \right)} \]
Section A (52 Marks)

Answer all questions in this section

1 (a) Simplify \( \frac{10x^3y^2}{35x^3y^4} + \frac{2x^2y^2}{7x^4y^2} \). [2]

(b) In a geometric progression, the third term is 16 and the fifth term is 4. Calculate
(i) the first term and the common ratio, [3]
(ii) the tenth term, [2]
(iii) the sum to infinity. [2]

2 (a) The determinant of matrix \( Q = \begin{pmatrix} 8 & 12 \\ x-4 & x \end{pmatrix} \) is 8. Find
(i) the value of \( x \), [2]
(ii) the inverse of \( Q \). [2]

(b) The Venn diagram below shows the optional subjects that all the Grade 10 learners at Kusambilila Secondary School took, in a particular year.

![Venn Diagram](image)

(i) Given that 12 learners took Music, find the value of \( x \). [2]
(ii) How many learners were in Grade 10 this particular year? [1]
(iii) Find the number of learners who took
(a) one optional subject only, [1]
(b) two optional subjects only. [1]
3. (a) Express \( \frac{6}{n-3} - \frac{5}{n-2} \) as a single fraction in its simplest form.

(b) In the diagram below, \( \overrightarrow{OA} = a, \overrightarrow{OB} = b \) and \( \frac{AC}{CB} = \frac{1}{2} \).

(i) Express in terms of \( a \) and/or \( b \)

(a) \( \overrightarrow{AB} \),

(b) \( \overrightarrow{AC} \),

(c) \( \overrightarrow{OC} \).

(ii) Given that \( M \) is the midpoint of \( OC \), show that \( \overrightarrow{AM} = \frac{1}{6}(b - 4a) \).

4. (a) (i) Construct a triangle \( JKL \) in which \( KL = 8 \text{cm}, KJ = 6 \text{cm} \) and \( JL = 10 \text{cm} \).

(ii) Measure and write angle \( JKL \).

(b) Within the triangle \( JKL \), draw the locus of points which are

(i) 5cm from \( J \),

(ii) 3cm from \( JL \),

(iii) equidistant from \( JK \) and \( JL \).

(c) A point \( Q \), within triangle \( JKL \), is such that it is greater than or equal to 5cm from \( J \), less than or equal to 3cm from \( JL \) and nearer to \( JK \) than to \( JL \). Indicate by shading the region in which \( Q \) must lie.
5  (a) Solve the equation $13 - 9x - 5x^2 = 0$, giving your answers correct to 2 decimal places. [5]

(b) Thirteen cubes of the same size numbered 1 to 13 are placed in a bag. If two cubes are drawn at random from the bag one after the other and not replaced, what is the probability that
   (i) both cubes are odd numbered, [2]
   (ii) only one is even numbered. [3]

6  (a) The gradient function of a curve is $y = 6x + 8$. Find the equation of the curve passing through the point $(1, 2)$. [3]

(b) The flow chart below shows the steps in calculating the volume of a solid given the base area $(A)$ and height $(h)$.

```
Start
   ↓
Enter A
   ↓
Is A < 0
   Yes "error message"
   No "A must be positive"
   ↓
Enter h
   ↓
Is h < 0
   Yes "error message"
   No "h must be positive"
   ↓
   V = A * h
   ↓
Display V
   ↓
Stop
```

Write the corresponding pseudocode for the flow chart given above. [5]
Section B [48 marks]

Answer any four questions in this section.

Each question in this section carries 12 marks.

7 Answer the whole of this question on a sheet of graph paper.

Mipando makes two types of chairs for sale; dining and garden. He intends to make at least 10 dining chairs and at least 20 garden chairs. He wants to make not more than 80 chairs altogether. The number of garden chairs must not be more than three times the number of dining chairs.

(a) Let \( x \) be the number of dining chairs and \( y \) the number of garden chairs. Write four inequalities to represent the information above. 

(b) Using a scale of 2 cm to represent 10 chairs on each axis, draw \( x \) and \( y \) axes for \( 0 \leq x \leq 80 \) and \( 0 \leq y \leq 80 \) respectively and shade the unwanted region to indicate clearly the region where the solution of the inequalities lie.

(c) Given that the profit on the sale of a dining chair is K80.00 and profit on a garden chair is K50.00, how many chairs of each type should Mipando make in order to maximize the profit?

(d) What is this maximum profit?

8 (a) In triangle ABC below, \( AC = 275 \) km, angle \( BAC = 125^\circ \) and angle \( ACB = 40^\circ \).

Calculate

(i) the distance \( BC \),

(ii) the area of triangle \( ABC \),

(iii) the shortest distance from \( A \) to \( BC \).

(b) Solve the equation \( 13 \cos \theta = 5 \) for \( 0 \leq \theta \leq 360^\circ \).

(c) Simplify \( \frac{2x^2 - 18}{x + 3} \).
9 (a) The table below shows the distribution of the ages of 30 football players at a school.

<table>
<thead>
<tr>
<th>Age (x) years</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>0</td>
<td>2</td>
<td>5</td>
<td>7</td>
<td>8</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

Calculate the standard deviation. [6]

(b) Answer this part of the question on a sheet of graph paper.

(i) Using the table above, copy and complete the relative cumulative frequency table below.

<table>
<thead>
<tr>
<th>Age (x) years</th>
<th>(\leq 10)</th>
<th>(\leq 11)</th>
<th>(\leq 12)</th>
<th>(\leq 13)</th>
<th>(\leq 14)</th>
<th>(\leq 15)</th>
<th>(\leq 16)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative Frequency</td>
<td>0</td>
<td>2</td>
<td>7</td>
<td>14</td>
<td>22</td>
<td>28</td>
<td>30</td>
</tr>
<tr>
<td>Relative cumulative frequency</td>
<td>0.00</td>
<td>0.07</td>
<td>0.23</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

[1]

(ii) Using a scale of 2cm to represent 1 unit on the x-axis for \(10 \leq x \leq 16\) and a scale of 2cm to represent 0.1 units on the y-axis for \(0.0 \leq y \leq 1.0\), draw a smooth relative cumulative frequency curve. [3]

(iii) Showing your method clearly, use your graph to estimate the 90th percentile. [2]

10 (a) The diagram below shows a frustum TQRS of a cone. [Take \(\pi = 3.142\)]

Given that \(US = 3\) cm, \(UV = 10\) cm and \(RV = 8\) cm, calculate its volume. [6]
(b) The points K, L and M are on the surface of the earth as shown in the diagram below.

[Take \( \pi \) as 3.142 and \( R = 6370 \text{km} \)]

(i) Find the difference in longitude between points K and L. [2]

(ii) Find, in kilometres, the distance

(a) LM, [2]

(b) KL. [2]
Study the diagram below and answer the questions that follow.

(a) An enlargement maps triangle ABC onto triangle \(A_1B_1C_1\). Find
(i) the centre of enlargement, [1]
(ii) the scale factor. [1]

(b) Triangle ABC is mapped onto triangle \(A_2B_2C_2\) by a single transformation. Describe fully this transformation. [3]

(c) Triangle ABC is mapped onto triangle \(A_3B_3C_3\) by a stretch. Find
(i) the matrix which represents this transformation, [3]
(ii) find the area scale factor. [1]

(d) A transformation matrix \[
\begin{pmatrix}
1 & 0 \\
2 & 1
\end{pmatrix}
\] maps triangle ABC onto triangle \(A_4B_4C_4\), not drawn on the diagram. Find the coordinates of \(A_4\), \(B_4\) and \(C_4\). [3]
12 (a) Answer this part of the question on a sheet of graph paper.

The values of $x$ and $y$ are connected by the equation $y = x^3 - 5x + 3$. Some corresponding values of $x$ and $y$ are given in the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-3$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$k$</td>
<td>$5$</td>
<td>$7$</td>
<td>$3$</td>
<td>$-1$</td>
<td>$1$</td>
<td>$15$</td>
</tr>
</tbody>
</table>

(i) Calculate the value of $k$. [1]

(ii) Using a scale of 2cm to 1 unit on the x-axis for $-3 \leq x \leq 3$ and 2cm to represent 5 units on the y-axis for $-10 \leq y \leq 20$, draw the graph of $y = x^3 - 5x + 3$. [3]

(iii) Use your graph to

(a) solve the equation $x^3 - 5x = 0$, [2]

(b) estimate the area bounded by the curve, $y = 3$ and $x = -2$. [3]

(b) Find the equation of the tangent to the curve $y = (2x + 3)^3$ at the point where $x = -1$. [3]